Impulse Response of 3-D Molecular Communication via Diffusion and Flow Channel with an Absorbing Receiver

Bayram Cevdet Akdeniz, H. Birkan Yilmaz, Ali Emre Pusane, Tuna Tugcu

Abstract—This work proposes an approach to find the channel impulse response (CIR) of a 3-D unbounded molecular communication channel in which molecules move under the effect of diffusion and flow. The main concept behind this approach is representing the equivalent distribution of molecules in terms of velocities has already been derived before. The derived analytical function has been compared with Monte Carlo simulation results for various flow velocities, and a fitted region in terms of velocities has been determined.

I. INTRODUCTION

For communication of nanomachines, molecular communication via diffusion (MCvD), which is based on encoding the intended information symbol by releasing molecules into a fluidic medium, has been proposed in the literature. The receiver in MCvD channels either acts as an absorber, which removes the incoming molecules from the environment [1], or as an observer which observes the incoming molecules without absorbing them [2]. The channel impulse responses (CIRs) for both types of receivers over diffusive molecules is available as presented in [3]. For all channel models in MCvD, one of the most fundamental problems is inter symbol interference (ISI) that occurs due to the random movement of molecules under the diffusion. In order to reduce ISI, flow is considered as a good solution in molecular communication. Since molecules move slowly due to the nature of the diffusion, flow in the channel may increase the speed of molecules and, this may potentially lead to an increase in the data rate. In [4], CIR of a 3-D unbounded medium with an observing receiver and an arbitrary flow is derived and an optimal receiver design is proposed for this channel. In [5], CIR of a 1-D medium with an absorbing receiver and an arbitrary flow is derived and communication strategies are proposed using CIR. In [6], using CIR, the optimal symbol interval is derived for reducing ISI. All these works show promising results by utilizing the effect of flow using CIR of corresponding Molecular Communication via Diffusion and Flow (MCvDaF) channels. Although the CIR of MCvDaF with a observer receiver in 3-D medium is derived in [4], the CIR of 3-D MCvDaF channel with an absorbing receiver has not been derived yet. Due to lack of the analytical formulation of the CIR, proposed methods that utilize flow can not be directly applied to the channels that have absorbing receiver. In this work, an approximation for deriving the CIR of the MCvDaF channel with a point transmitter and a fully absorbing spherical receiver in a 3-D is proposed.

II. SYSTEM MODEL

The system model of MCvD and MCvDaF channels are depicted in Fig. 1(a) and 1(b), respectively. In these figures, without loss of generality, a point transmitter placed at \((r_t,0,0)\) releases molecules to convey its information to a spherical receiver of radius \(r_r\) placed at the origin \((0,0,0)\). The released molecules diffuse through the fluidic medium and the receiver absorbs these molecules, if they move to the vicinity of its receptors. The movement of molecules depends on the Gaussian distribution of molecules when the flow is zero for any instant \(t\). Although the flow is changing around an absorbing receiver as stated in [7], in this paper we ignore the effect of the receiver on the flow to use a simpler model and left the consideration of this effect as a future work. Let \(r = [\Delta x \quad \Delta y \quad \Delta z]\) be the displacement vector of a molecule for a period \(\Delta t\). Then, as can be seen in [3], this vector can be represented considering the diffusion and flow terms as

\[
\Delta k = \Delta t v_k + N(0, 2D\Delta t),
\]

where \(k \in \{x, y, z\}\) and \(N\) represents the Gaussian distribution. Assuming that the transmitter, which is \(r_t = r_0\) away from the center of the receiver, releases a molecule to environment at \(t = 0\), the joint probability density of this molecule with respect to time \((t)\) and distance \((r)\) is presented in [8] as

\[
p(r,t|r_t = r_0, v = 0) = \frac{1}{4\pi r_0 r V} e^{-\frac{1}{4D} \left(\frac{(r-r_0)^2}{4D} - \frac{(r+r_0-2r_e)^2}{4D}\right)}.
\]

Once \(p(r,t|r_t = r_0, v = 0)\) is obtained, the channel impulse response that determines the density of the absorbed molecules by the receiver with respect to time, \(n_{hit}(t|r_t = r_0, v = 0)\), has also been derived in [8] as

\[
n_{hit}(t|r_t = r_0, v = 0) = 4\pi r_e^2 \frac{1}{v} \frac{\partial}{\partial t} p(r, t|r_t)
\]

\[
= r_e \frac{1}{V (r_0 - r_e)^2} \frac{(r_0 - r_e)^2}{4D}.
\]

III. CHANNEL IMPULSE RESPONSE WHEN \(v = [v \ 0 \ 0]\)

When the flow only involves one component whose direction is parallel to the line passing through the transmitter and the center of the receiver, as shown in Fig. 1(b), CIR can be derived as follows: Fig. 1(a) depicts the distribution of molecules when the flow is zero for any instant \(t\). The distribution of molecules when the flow is available is plotted

\[
\text{Impulse Response of 3-D Molecular Communication via Diffusion and Flow Channel with an Absorbing Receiver}
\]

Bayram Cevdet Akdeniz, H. Birkan Yilmaz, Ali Emre Pusane, Tuna Tugcu

Abstract—This work proposes an approach to find the channel impulse response (CIR) of a 3-D unbounded molecular communication channel in which molecules move under the effect of diffusion and flow. The main concept behind this approach is representing the equivalent distribution of molecules in terms of velocities has already been derived before. The derived analytical function has been compared with Monte Carlo simulation results for various flow velocities, and a fitted region in terms of velocities has been determined.

I. INTRODUCTION

For communication of nanomachines, molecular communication via diffusion (MCvD), which is based on encoding the intended information symbol by releasing molecules into a fluidic medium, has been proposed in the literature. The receiver in MCvD channels either acts as an absorber, which removes the incoming molecules from the environment [1], or as an observer which observes the incoming molecules without absorbing them [2]. The channel impulse responses (CIRs) for both types of receivers over diffusive molecules is available as presented in [3]. For all channel models in MCvD, one of the most fundamental problems is inter symbol interference (ISI) that occurs due to the random movement of molecules under the diffusion. In order to reduce ISI, flow is considered as a good solution in molecular communication. Since molecules move slowly due to the nature of the diffusion, flow in the channel may increase the speed of molecules and, this may potentially lead to an increase in the data rate. In [4], CIR of a 3-D unbounded medium with an observing receiver and an arbitrary flow is derived and an optimal receiver design is proposed for this channel. In [5], CIR of a 1-D medium with an absorbing receiver and an arbitrary flow is derived and communication strategies are proposed using CIR. In [6], using CIR, the optimal symbol interval is derived for reducing ISI. All these works show promising results by utilizing the effect of flow using CIR of corresponding Molecular Communication via Diffusion and Flow (MCvDaF) channels. Although the CIR of MCvDaF with a observer receiver in 3-D medium is derived in [4], the CIR of 3-D MCvDaF channel with an absorbing receiver has not been derived yet. Due to lack of the analytical formulation of the CIR, proposed methods that utilize flow can not be directly applied to the channels that have absorbing receiver. In this work, an approximation for deriving the CIR of the MCvDaF channel with a point transmitter and a fully absorbing spherical receiver in a 3-D is proposed.

II. SYSTEM MODEL

The system model of MCvD and MCvDaF channels are depicted in Fig. 1(a) and 1(b), respectively. In these figures, without loss of generality, a point transmitter placed at \((r_t,0,0)\) releases molecules to convey its information to a spherical receiver of radius \(r_r\) placed at the origin \((0,0,0)\). The released molecules diffuse through the fluidic medium and the receiver absorbs these molecules, if they move to the vicinity of its receptors. The movement of molecules depends on the Gaussian distribution of molecules when the flow is zero for any instant \(t\). Although the flow is changing around an absorbing receiver as stated in [7], in this paper we ignore the effect of the receiver on the flow to use a simpler model and left the consideration of this effect as a future work. Let \(r = [\Delta x \quad \Delta y \quad \Delta z]\) be the displacement vector of a molecule for a period \(\Delta t\). Then, as can be seen in [3], this vector can be represented considering the diffusion and flow terms as

\[
\Delta k = \Delta t v_k + N(0, 2D\Delta t),
\]

where \(k \in \{x, y, z\}\) and \(N\) represents the Gaussian distribution. Assuming that the transmitter, which is \(r_t = r_0\) away from the center of the receiver, releases a molecule to environment at \(t = 0\), the joint probability density of this molecule with respect to time \((t)\) and distance \((r)\) is presented in [8] as

\[
p(r,t|r_t = r_0, v = 0) = \frac{1}{4\pi r_0 r V} e^{-\frac{1}{4D} \left(\frac{(r-r_0)^2}{4D} - \frac{(r+r_0-2r_e)^2}{4D}\right)}.
\]

Once \(p(r,t|r_t = r_0, v = 0)\) is obtained, the channel impulse response that determines the density of the absorbed molecules by the receiver with respect to time, \(n_{hit}(t|r_t = r_0, v = 0)\), has also been derived in [8] as

\[
n_{hit}(t|r_t = r_0, v = 0) = 4\pi r_e^2 \frac{1}{v} \frac{\partial}{\partial t} p(r, t|r_t)
\]

\[
= r_e \frac{1}{V (r_0 - r_e)^2} \frac{(r_0 - r_e)^2}{4D}.
\]

III. CHANNEL IMPULSE RESPONSE WHEN \(v = [v \ 0 \ 0]\)

When the flow only involves one component whose direction is parallel to the line passing through the transmitter and the center of the receiver, as shown in Fig. 1(b), CIR can be derived as follows: Fig. 1(a) depicts the distribution of molecules when the flow is zero for any instant \(t\). The distribution of molecules when the flow is available is plotted
in Fig. 1(b) and it is the $vt$ shifted version of the distribution of the molecules in Fig. 1(a). Furthermore, the distribution of molecules in Fig. 1(b) is approximated by the distribution of molecules for the case when the transmitter is placed $vt$ closer to the receiver in the medium without flow, which is shown in Fig 1(c). Mathematically,

$$p(r, t | r_0 = r_v, v = [v \ 0 \ 0]) = p(r, t | r_0 = r_v - vt, \bar{v} = 0)$$

Then, using (3) and (4), $n_{hit}(t | r_0 = r_v, v = [v \ 0 \ 0])$ can be approximated as

$$n_{hit}(t | r_0 = r_v, v = [v \ 0 \ 0]) = 4\pi r_v^2 \frac{r_v}{t} p(r, t | r_0 = r_v - vt) |_{r=r_v}$$

$$= \frac{r_v}{r_v - vt \sqrt{4\pi Dt}} \frac{(r_0 - vt - r_v)^2}{4Dt}.$$ 

IV. SIMULATION RESULTS

The channel impulse response obtained in (5) has been tested using Monte Carlo simulations. In these simulations, $10^6$ molecules are released and their positions are updated according to the dynamics of diffusion and flow. The time increment, $\Delta t$, is chosen as $10^{-4} \text{s}$ and the simulations are conducted for a total duration of 4s. The derived and simulated results have been tested for velocities differing in the region $10^{-6} \sim 10^{-5} \text{m/s}$. The comparison between analytical and simulated results have been evaluated using the MATLAB’s goodness of fit (GoF) toolbox and the results are presented in Fig. 2. As can be deduced from this figure, our proposed channel impulse response function fits very well with simulation results until a velocity value that is proportional to $r_0$. For higher velocities (especially higher than $r_0 - r_v$), our proposed CIR function does not fit with simulation results which is expected. For higher velocities, change in the flow around the receiver may not be ignored, plus the distribution of molecules plotted in the Fig.1(b) may not be valid since some of the molecules can be absorbed by the receiver due to the shift caused by the flow. Hence (5) will become an overestimated distribution for higher velocities as time increases.

V. CONCLUSION AND FUTURE WORKS

In this work, we have derived an approximation for the CIR of a MCvDaF channel that involves an absorbing receiver using the CIR of MCvD channel. We have tested our proposed CIR function using Monte Carlo simulation and determined its valid region in terms of velocity. For future work, our aim is to extend CIR function estimation for higher velocity values and arbitrary directions using heuristic approaches and consider also more realistic channels as vessel like environments.

REFERENCES


