Controlled Release for Diffusive Mobile Molecular Communication Systems

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***Abstract*—In diffusive mobile molecular communication systems, both the signaling molecules and the transceivers may diffuse over time. Thus, the distance between the transceivers constantly changes and, in general, is not always available for adapting the receiver. Assuming that the distance between the transceivers is known only at the start of the transmission bit frame, we design a statistically optimal allocation of the number of molecules available for controlled released by the transmitter for each bit using on-off keying modulation given a fixed budget of molecules. We show that the system performance in terms of the bit error rate is significantly improved compared to uniform allocation of molecules to each bit.**

I. Introduction

In this work, we study a diffusive molecular communication (MC) system with mobile transceivers, where the signaling molecules and the transceivers follow Brownian motion [1]. Diffusive mobile MC systems are a relevant scenario for nano-networks where tiny sensors diffuse in the environment and communicate with each other via molecules. For a simple MC system using on-off keying and threshold detection, the distance between the transceivers at the time of release has to be known for reliable communication design. However, this information may not always be available in a diffusive mobile MC system due to the random movements of the transceivers. In particular, the distance between the transceivers can only be available at the start of each transmitted packet, i.e., each information bit frame. Given a fixed number of molecules available for transmission in each frame and keeping the complexity of the receiver low, the question is whether there is a clever transmitter design that guarantees a certain error rate for each bit in the frame? In this work, we design a controlled-release profile at the transmitter specifying the optimal number of molecules available for each bit in a frame, and show the improvement on the bit error rate (BER) compared to a standard uniform release, i.e., release of an equal number of molecules available for every bits. Note that the optimal design framework can be applied to every frame but the values of the optimal numbers of released molecules can be different depending on the distance between the transceivers at the start of the frame. In the following, we first characterize the system statistically and derive the BER as a function of the number of released molecules. Then, we optimize the controlled-release profile in terms of the BER and show numerical results. The numerical results assume an absorbing receiver but the framework developed in this work can be applied for both absorbing and transparent receivers.

II. System Model

We consider an unbounded three-dimensional environment comprising one mobile spherical transparent transmitter, denoted by Tx, one mobile spherical receiver, denoted by Rx, and the signaling molecules of type X. The movements of Tx, Rx, and the X molecules are assumed to follow Brownian motion with diffusion coefficients D_{Tx}, D_{Rx}, and D_X, respectively. We assume that the time-varying distance between the centers of Tx and Rx at time t, denoted by $r(t)$, is only known at the start of each transmission of an I-bit frame in a time interval of length T. We consider an on-off keying system with multi-frame communication. For an arbitrary bit frame, let b_i, $i \in \{1, \ldots, I\}$, denote the i-th bit in the bit frame. For the i-th bit in a frame, at time t_i, Tx releases α_i or no molecules to transmit bit 1 or bit 0, respectively. Let $A = \sum_{i=1}^{I} \alpha_i$ denote the total number of molecules available for transmission in a bit frame. We assume that symbols 0 and 1 are transmitted independently and with equal probability, thus the probability of transmitting $\tilde{b}_i$ is $\Pr(\tilde{b}_i) = 1/2$, where $\tilde{b}_i \in \{0, 1\}$ is a realization of b_i.

Let $h(t, \tau)$ denote the channel impulse response (CIR). For an absorbing receiver, $h(t, \tau)$ is defined as the probability that a signaling molecule is absorbed during time τ after its release at time t at the center of Tx. For a transparent receiver, $h(t, \tau)$ is the observation probability of a molecule inside the volume of the transparent receiver at time τ after its release at time t at the center of Tx. For a fixed distance $r(t)$, expressions for the CIR of passive and absorbing receivers have been reported in [2] and [3], respectively. Since $r(t)$ is a random variable in this work, $h(t, \tau)$ and any functions of $h(t, \tau)$ depend on the distribution of $r(t)$. Here, we consider the number of molecules X absorbed at Rx during time τ after the transmission of the i-th bit at Tx at time t_i, denoted by g_i, as the received signal. It has been shown in [3] that for α_i molecules released at t_i, the number of received molecules follows a Binomial distribution and can be accurately approximated by a Gaussian distribution when α_i is large, which we will assume here. In this work, we focus on the effect of the transceivers' movements on the system performance and design an optimal release profile for the transmitter to compensate for the transceivers' movements. To this end, we assume the bit interval to be long enough such that most of the molecules are captured by or move far away from the Rx before the transmission of the following bit and thus there is no significant inter-symbol interference (ISI). In particular, for simplicity, we model the small number of absorbed molecules originating from the previous bits and from external noise sources in the environment as a Gaussian background noise with mean and variance equal to n. Thus, we have

$$g_i \sim \mathcal{N}\left(\mu_{i,\tilde{b}_i}, \sigma_{i,\tilde{b}_i}\right) \text{ for } b_i = \tilde{b}_i. \tag{1}$$

where $\mu_{i,0} = \sigma_{i,0}^2 = n$, $\mu_{i,1} = \alpha_i h(r_i) + n$, $\sigma_{i,1}^2 = \alpha_i h(r_i)(1 - h(r_i)) + n$. For brevity, we use $h(r_i)$ for $h(t_i, \tau)$, and r_i for $r(t_i)$.

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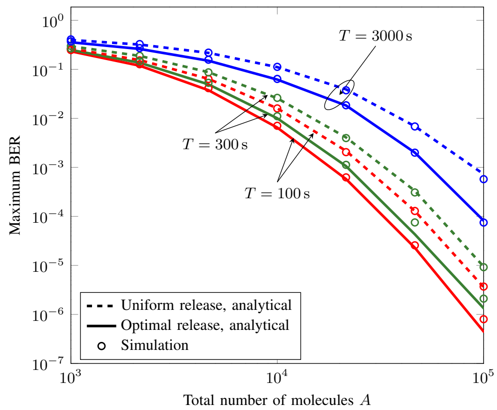

Fig. 1. Maximum BER as a function of A with uniform and optimal release.

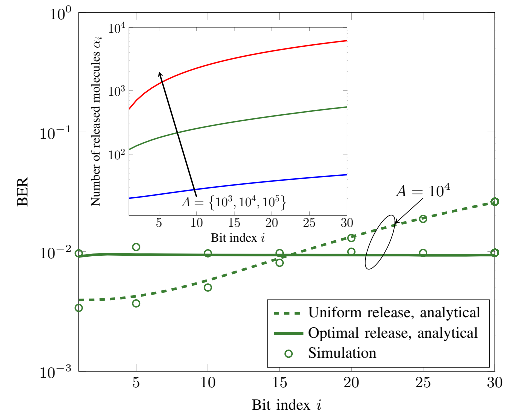

Fig. 2. BER for each bit in a frame for uniform and optimal release for $A = 10^4$ and $T = 300\,\mathrm{s}$. The inset shows the number of released molecules for each bit with the optimal release for $A = \{10^3, 10^4, 10^5\}$ and $T = 300\,\mathrm{s}$.

We consider a simple threshold detector, where the received signal g_i is compared with a detection threshold, denoted by ξ, in order to determine the detected bit $\hat{b}_i$ as follows

$$\hat{b}_i = \begin{cases} 1 \text{ if } g_i > \xi, \\ 0 \text{ if } g_i \leqslant \xi. \end{cases} \tag{2}$$

Given the assumption of no ISI and $\Pr(\tilde{b}_i) = 1/2$, from (1) and (2), the expected error probability of the i-th bit, denoted by $P_\mathrm{b}(b_i)$, can be simplified as [1, Eq. 12]

$$P_\mathrm{b}(b_i) = \frac{1}{2} - \frac{1}{4}\mathrm{erf}\left(\frac{\xi - n}{\sqrt{2n}}\right) + \frac{1}{4}\int_0^\infty f(r_i)\mathrm{erf}\left(\gamma_i(\xi, \alpha_i)\right) \mathrm{d}r_i, \tag{3}$$

where $f(r_i)$ is the probability density function of r_i, given in [4, Eq. 6] with D_Tx replaced by $D_\mathrm{Tx} + D_\mathrm{Rx}$, $\mathrm{erf}(\cdot)$ is the error function, and $\gamma_i(\xi, \alpha_i) = \frac{\xi - \mu_{i,1}}{\sigma_{i,1}\sqrt{2}} = \frac{\xi - (\alpha_i h(r_i) + n)}{\sqrt{2(\alpha_i h(r_i)(1 - h(r_i)) + n)}}$.

III. Problem Formulation

Our aim is to optimize the number of molecules available for each bit, α_i, in order to minimize the maximum error rate of the bits in a frame, given a total number of molecules, denoted as A, that can be used in a frame:

$$\min_{\boldsymbol{\alpha}} \max_i \{P_\mathrm{b}(b_i)\} \qquad \text{s.t.} \sum_{i=1}^{I} \alpha_i = A, \tag{4}$$

where $\boldsymbol{\alpha}$ is the vector with elements α_i. For a given threshold $\xi^\star$, we can re-express (4) based on (3) as

$$\min_{\boldsymbol{\alpha}} \max_i \left\{\int_0^\infty f(r_i)\mathrm{erf}\left(\gamma_i\left(\xi^\star, \alpha_i\right)\right) dr_i\right\} \text{ s.t.} \sum_{i=1}^{I} \alpha_i = A. \tag{5}$$

For $\xi < \mu_{i,1}$, we are able to prove that the cost functions in (5) is convex in $\boldsymbol{\alpha}$. Then, (5) can be readily solved by existing numerical solvers and the global optimum can be obtained. Note that $\xi < \mu_{i,1}$ is intuitively satisfied for an optimal design since the threshold should not exceed the mean of the received signal for a bit 1 transmission. Otherwise, there would be a high error rate, on average, when transmitting bit 1. In particular, one way to choose ξ in a systematic manner so that $\xi < \mu_{i,1}$ is by minimizing $\max_i \{P_\mathrm{b}(b_i)\}$ over ξ for a uniform release. In this way, from (3) we can conveniently obtain ξ by solving

$$\min_{\xi} \max_i \left\{\int_0^\infty f(r_i)\mathrm{erf}\left(\gamma_i(\xi, \alpha_i)\right) dr_i - \mathrm{erf}\left(\frac{\xi - n}{\sqrt{2n}}\right)\right\}, \tag{6}$$

where $\alpha_i = A/I$. For $\xi < \mu_{i,1}$, we are able to prove that the cost functions in (6) is convex in ξ and thus the global optimum of ξ can be easily obtained by a numerical solver.

IV. Numerical Examples

In numerical examples, we apply the following parameters: $D_\mathrm{Tx} = 0.1\,\mu\mathrm{m}^2/\mathrm{s}$, $D_\mathrm{Rx} = 0.1\,\mu\mathrm{m}^2/\mathrm{s}$, $D_X = 80\,\mu\mathrm{m}^2/\mathrm{s}$, $a_\mathrm{rx} = 1\,\mu\mathrm{m}$, $r_0 = 10\,\mu\mathrm{m}$, $I = 30$, $n = 1$, $\tau = T/I$, and $T = \{100, 300, 3000\}\,\mathrm{s}$. We adopt Monte-Carlo simulation by averaging the BER over a large number of independent realizations of the transceivers' movements during a bit frame.

Fig. 1 shows the maximum BER within a frame for the proposed release design and a uniform release for different A and T. As can be observed, the proposed optimal release profile leads to significant performance improvements compared to uniform release, especially for large A. For example, for $A = 10^5$ and $T = 3000$s, the maximum BER is reduced by a factor of 8 for optimum release compared to uniform release.

In Fig. 2, we plot the BER as a function of bit index i in one frame for uniform and optimal release for $A = 10^4$ and $T = 300\,\mathrm{s}$. We see in the figure that the optimal release achieves a lower maximum BER compared to the uniform release. We also observe that optimal release achieves approximately the same BER for each bit which highlights the effectiveness of our proposed design. In the inset of Fig. 2, we show the optimal number of released molecules for each bit in a frame for $T = 300\,\mathrm{s}$. To achieve a lower maximum BER, fewer molecules are released at the beginning of the frame and the number of released molecules gradually increases during the frame. This is intuitive since we need more molecules to compensate for the on-average-increasing distance between the transceivers.

References

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